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Modeling the Resilient Modulus of Subgrade Soils With a Four-Parameter Constitutive Equation

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Abstract

A new constitutive model for the resilient modulus (M_R) of subgrade soils was developed in this study based on existing models. The general form of three-parameter model which is common in many constitutive equations for M_R was extended here to have four parameters. The proposed model relates M_R with the bulk stress, confining pressure and deviatoric stress of the soil using four coefficients. The performance of the model was tested by fitting it to an M_R test data obtained from the long-term pavement performance database. A multi-variable nonlinear curve fitting was done using the SciPy library. Results showed that the model has a very good fit to the data with coefficient of determination, root mean square error and mean absolute error values of 0.94, 2.20 and 1.76, respectively. The results of the proposed model were also compared with the bulk stress model and the universal model, which is currently used by the mechanistic-empirical pavement design guide (MEPDG), and obtained to be generally better than both models. The proposed model could potentially be a good alternative to the existing constitutive models if methods for the determination of the k-coefficients could be developed.

Keywords

Resilient modulus, pavement subgrade, constitutive models, curve fitting

Statements and Declarations

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Author Contributions

Ayenew Yihune Demeke: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data Curation, Writing - Original Draft, Visualization. **Constantine Sachpazis:** Resources, Writing - Review & Editing, Supervision. **Eleyas Assefa:** Writing - Review & Editing, Supervision. **Lysandros Pantelidis:** Writing - Review & Editing, Supervision.

1. Introduction

Resilient modulus (M_R) is a fundamental soil property used to characterize pavement materials during design. It is currently used as the main parameter to characterize subgrade soils during the design of flexible pavements (AASHTO 1986, 1993, 2002). The direct way of determining M_R of a soil is using a repeated load triaxial (RLT) test (AASHTO 2003). As M_R is dependant on the stress state of the soil, this test is generally done under a controlled confining and deviatoric stress conditions.

Although the results of RLT are basically reliable, it may not be easy to do the test due to the fact that the test is relatively complex and needs a sophisticated laboratory facility and skilled technician (Heidaripناه et al. 2017; Saha et al. 2018; Khasawneh and Al-jamal 2019). As this could not be accessible for many laboratories established for routine testing in construction areas, other alternative methods were proposed by different researchers and design standards have also stated recommendations of estimating M_R based on these researches. Most of these alternative methods are predictive models which attempt to estimate M_R from other simple soil properties through empirical relationships. These predictive methods can generally be categorized as, correlative equations and constitutive equations.

Correlation equations relate M_R with other soil properties which can be determined from laboratory or field tests with relative ease. One such equation is, for example, the one proposed by Powell et al. (1984) and included in the mechanistic-empirical pavement design guide (MEPDG) by which M_R is estimated from CBR (Eq. 1). Other similar equations were proposed by several researchers (Mohammad et al. 2002, 2007; AASHTO 2003; Rao et al. 2012; Hamed Mousavi et al. 2018;

Sadrossadat et al. 2018, 2020; Ghorbani et al. 2020). In addition to simple equations mentioned above, several researchers have developed more advanced predictive models using different machine learning algorithms including neural networks, support vector machines and ensemble algorithms (Ozsahin and Oruc 2008; Pourtahmasb et al. 2015; Sadrossadat et al. 2016; Heidariapanah et al. 2017; Saha et al. 2018; Khasawneh and Al-jamal 2019; Ren et al. 2019; Ghorbani et al. 2020; Hanandeh et al. 2020; Heidarabadizadeh et al. 2021; Zou et al. 2021).

$$M_R(psi) = 2554(CBR)^{0.64} \quad (1)$$

On the other hand, various constitutive equations were proposed throughout the recent past with the intention of developing a constitutive relationship between M_R and the stress state in the soil. The bulk stress model (Eq. 2) was in common use by most pavement design engineers before the universal model (Eqn. 3) was proposed by the national cooperative highway research program (NCHRP) project 1-28A (Santha 1994; Witczak 2003; NCHRP 2008; Murana 2018).

$$M_R = k_1 P_a \left(\frac{\theta}{P_a} \right)^{k_2} \quad (2)$$

$$M_R = k_1 P_a \left(\frac{\theta}{P_a} \right)^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \quad (3)$$

Several other equations were also proposed by different researchers. Most of these constitutive equations follow the general form of a three-parameter model shown in Eq. (4) (NCHRP 2008; Murana 2018). Some of these equations are summarized in Table 1.

$$M_R = k_1 P_a [f(c)]^{k_2} [g(s)]^{k_3} \quad (4)$$

Table 1: Constitutive equations for the prediction of M_R

Researcher	Model	Remark
Rahim and George (2007)	$M_R = k_1 P_a \left(1 + \frac{\sigma_d}{1 + \sigma_c} \right)^{k_2}$	For fine grained soils
	$M_R = k_1 P_a \left(1 + \frac{\theta}{1 + \sigma_d} \right)^{k_2}$	For coarse grained soils
Kim (2004)	$M_R = k_1 \left(\frac{P_a \cdot \sigma_{oct}}{\tau_{oct}^2} \right)^{k_2}$	For fine grained soils

Ni et al. (2002)	$M_R = k_1 P_a \left(\frac{\sigma_3}{P_a} + 1 \right)^{k_2} \left(\frac{\sigma_d}{P_a} + 1 \right)^{k_3}$
Puppala et al. (1996)	$M_R = k_1 P_a \left(\frac{\sigma_3}{P_a} \right)^{k_2} \left(\frac{\sigma_d}{P_a} \right)^{k_3}$
Witczak and Uzan (1988)	$M_R = k_1 P_a \left(\frac{\theta}{P_a} \right)^{k_2} \left(\frac{\tau_{oct}}{P_a} \right)^{k_3}$
Uzan (1985)	$M_R = k_1 (\theta)^{k_2} (\sigma_d)^{k_3}$
Moossazadeh and Witczak (1981)	$M_R = k_1 (\sigma_d)^{k_2}$
Seed et al. (1967)	$M_R = k_1 \left(\frac{\theta}{P_a} \right)^{k_2}$

As can be seen, the mentioned constitutive equations contain k-coefficients that relate M_R to the stress state. These coefficients are studied to be dependent on the inherent properties of the soil and various prediction mechanisms for these coefficients were proposed by different researchers (Yau and Quintus 2002; George 2004; Elias and Titi 2006; Malla and Joshi 2008; Nazzal and Mohammad 2010; Zhou et al. 2014; Sadrossadat et al. 2016; Saha et al. 2018; Kim et al. 2019).

The aim of this study was to develop a new model for M_R based on the existing models and testing its performance on a relatively large set of M_R test data obtained from the long-term pavement performance (LTPP) database. A model with four coefficients were proposed and tested for its performance. The performance of the proposed model was also compared with the bulk stress and universal constitutive equations.

2. Materials and Methods

The purpose of this paper is to evaluate the performance of a new model proposed for the relationship between M_R and the stress state in the soil by fitting the model to a data. For this purpose, M_R test data was extracted from the LTPP database. Curve fitting was done using the SciPy library of the Python programming language, which is capable of fitting nonlinear curves for single and multi-variable scenarios. Performance evaluation was done using coefficient of determination (R^2), root mean square error (RMSE) and mean absolute error (MAE). The

evaluated performance was then compared with the two widely used models: the universal model and the bulk stress model.

2.1. M_R Model

In this study, a new model was first proposed by extending the three-parameter model given in Eq. (4) to have the form given in Eq. (5).

$$M_R = k_1 P_a [f(\theta)]^{k_2} [g(c)]^{k_3} [h(d)]^{k_4} \quad (5)$$

Where $f(\theta)$ is a function of bulk stress, $g(c)$ is a function of confining pressure, $h(d)$ is a function of deviatoric stress and k_1 , k_2 , k_3 and k_4 are constants. Based on this modified form, the model given in Eqn. (6) was proposed and its performance was checked based on a large collection of M_R test data.

$$M_R = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\sigma_3}{P_a}\right)^{k_3} \left(\frac{\sigma_d}{P_a}\right)^{k_4} \quad (6)$$

2.2. M_R Test Data

M_R data of unbound pavement materials containing a total of 153,702 records was extracted from the LTPP database. In order to use these data for the desired purpose, data preprocessing work was done using the Python library Pandas. This generally included two major works: removing unnecessary data and grouping of tests done on a unique sample.

As the first step here, columns that were not necessary for the study were removed. It was then observed that some records (rows), although only very small fraction of the total data, contained null values in some columns or have M_R values of zero. These records were removed as the missing values will have problems during model development and M_R couldn't practically be zero. The next step done was removing duplicate data. This had a significant change in the number of records, as the extracted data had many duplicates intentionally done for other purposes. The final step done here was filtering subgrade data and removing the rest. This was done because the data also contained records of base and sub base materials.

Grouping was done to cluster the data based on the sample on which the tests are done. The data generally contained results of tests done on various soil samples obtained from different locations

in the United States and Canada. Fifteen tests were done on most samples by varying the stress state; each test having different combinations of σ_3 and σ_d . The purpose of the grouping work was therefore to bring together these test results, so that each group will contain tests on a unique sample. This grouping process generated a total of 1794 groups, which intern meant that the data contained tests done on 1794 samples. But it was noticed that only few tests were done on some of the samples and, therefore, samples with less than 15 tests were discarded to avoid potential shortage of data points during curve fitting. Fifty-three groups were removed by this process and the total number of groups left were 1741. A typical group is given in Table 2.

Table 2: Typical group containing results of M_R test on a sample

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1005	53	1	2	1	TP1	BS93	41.3	13.8	2	1	64
1005	53	1	2	1	TP1	BS93	41.3	68.9	2	1	56
1005	53	1	2	1	TP1	BS93	27.6	41.3	2	1	49
1005	53	1	2	1	TP1	BS93	27.6	55.1	2	1	48
1005	53	1	2	1	TP1	BS93	13.8	13.8	2	1	46
1005	53	1	2	1	TP1	BS93	13.8	27.6	2	1	41
1005	53	1	2	1	TP1	BS93	41.3	27.6	2	1	63
1005	53	1	2	1	TP1	BS93	13.8	68.9	2	1	41
1005	53	1	2	1	TP1	BS93	13.8	41.3	2	1	40
1005	53	1	2	1	TP1	BS93	13.8	55.1	2	1	40
1005	53	1	2	1	TP1	BS93	27.6	13.8	2	1	55
1005	53	1	2	1	TP1	BS93	41.3	41.3	2	1	59
1005	53	1	2	1	TP1	BS93	41.3	55.1	2	1	56
1005	53	1	2	1	TP1	BS93	27.6	27.6	2	1	52
1005	53	1	2	1	TP1	BS93	27.6	68.9	2	1	48

Where:

C1	SHRP_ID	C2	STATE_CODE
C3	LAYER_NO	C4	TEST_NO
C5	FIELD_SET	C6	LOC_NO

C7	SAMPLE_NO	C8	CON_PRESSURE
C9	NOM_MAX_AXIAL_STRESS	C10	CONSTRUCTION_NO
C11	LAYER_TYPE	C12	RES_MOD_AVG

2.3. Curve Fitting

As mentioned above, multi-variable non-linear curve fitting was done using the SciPy library. The curve fitting procedure generally included fitting test results in each group to the model and determining the model coefficients (k_1 , k_2 , k_3 and k_4). Prediction of M_R was then made using the model with the computed coefficients. The results were then compared with the measured values of M_R and the performance of the model on each group was evaluated based on the R^2 , RMSE and MAE values. The final performance of the model was taken to be the average of the model performance values on the 1741 groups mentioned in section 2.2 above. The same procedure was used to compute performances of the universal model and the bulk stress model and results were compared with the proposed model.

3. Results

This section contains the results of the multi-variable nonlinear curve fitting process done to compute the performance of the proposed new model for M_R . As discussed in the previous section, curve fitting was done on each of the collected 1741 groups of M_R test data. Table 3 contains the coefficients of the proposed model on the first five groups.

Table 3: Model coefficients on the first five groups

Group	k_1	k_2	k_3	k_4
1	1.360	-0.125	0.178	-0.017
2	0.789	0.801	-0.221	-0.387
3	0.573	0.070	0.240	-0.054
4	0.995	-0.066	0.262	-0.019
5	5.080	-1.608	1.154	0.497

The distributions of the computed k-coefficients of all groups are as shown in Figure 1, with an average values of 1.180, 0.029, 0.249 and -0.069 for k_1 , k_2 , k_3 and k_4 , respectively. The standard deviation values are 1.833, 0.452, 0.246 and 0.192 for k_1 , k_2 , k_3 and k_4 , respectively.

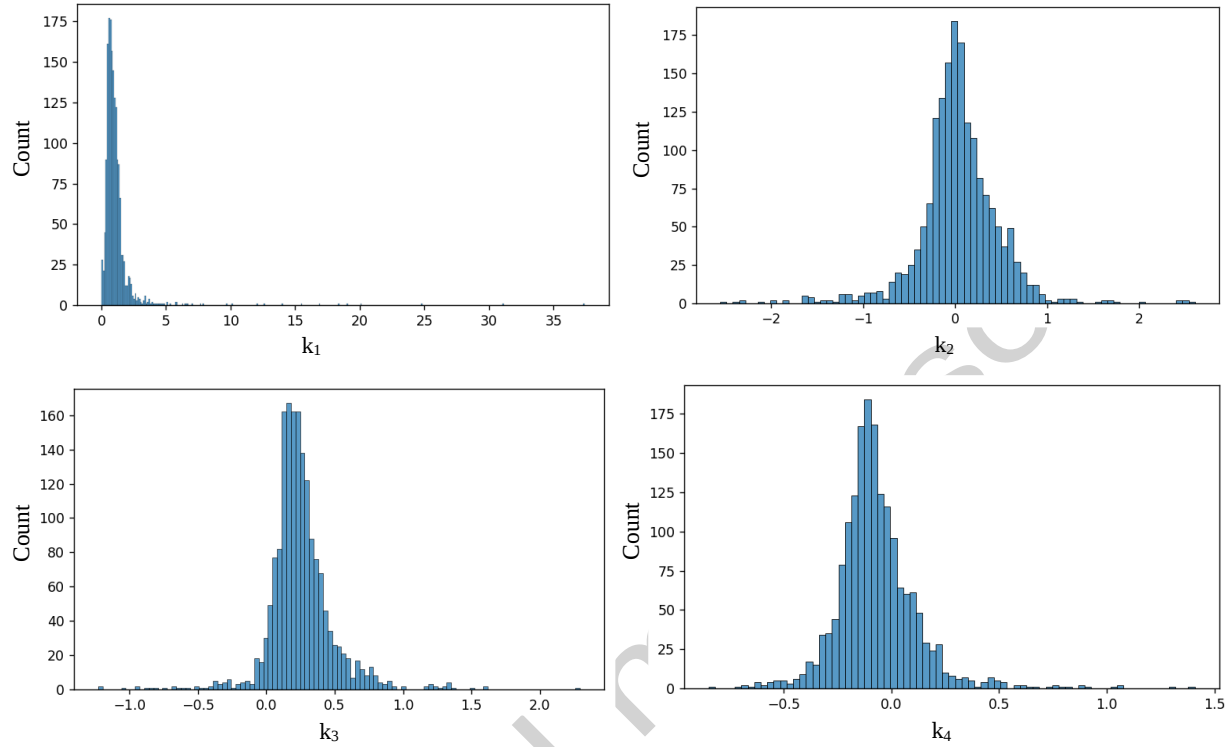


Figure 1: Histogram of k-coefficients

The R^2 , MAE and RMSE values for the first five groups is given in Table 4. Results of the universal model and bulk stress model are also given for comparison. As can be seen, the proposed model has generally a better performance for most of the groups. The universal model produced comparable results while the bulk stress model has relatively poor performance.

Table 4: R^2 , MAE and RMSE values for the first five groups

Proposed model			Universal model			Bulk stress model		
R^2	RMSE	MAE	R^2	RMSE	MAE	R^2	RMSE	MAE
0.90	1.97	1.51	0.92	1.78	1.43	0.13	5.71	4.93
0.88	11.96	8.98	0.73	17.99	14.65	0.20	31.11	22.44
0.97	0.96	0.85	0.93	1.53	1.24	0.59	3.72	3.17
0.97	1.31	1.15	0.90	2.25	1.81	0.51	5.06	4.22
0.67	3.48	2.85	0.07	5.89	4.59	0.06	5.92	4.74

As mentioned above, the final performance of the model was taken to be the average of the performances on each group. Table 5 shows the average values of R^2 , RMSE and MAE for the proposed model, universal model and bulk stress model. The result shows that the proposed model was better in fitting the data. The universal model, which is currently in use in the MEPDG, also produced good results while the results of the bulk stress model can be taken to be poor.

Table 5: Average R^2 , RMSE and MAE values of the constitutive models

Constitutive Equation	R^2	RMSE	MAE
Proposed model	0.94	2.20	1.76
Universal model	0.90	2.67	2.15
Bulk stress model	0.42	7.47	6.20

4. Conclusion

A new constitutive model was proposed in this study and the proposed model was tested using M_R test data. Comparison of performance was also made between the proposed model, the bulk stress model and the universal model. Based on the results obtained, the following conclusions were made.

- A nonlinear curve fitting analysis on M_R test data showed that the proposed model produced a good fit to the data with R^2 , RMSE and MAE values of 0.94, 2.20 and 1.76, respectively.
- Comparison of the proposed model with the bulk stress model and universal model showed that the proposed model has a better performance than both models. The resulting R^2 values were 0.94, 0.90 and 0.42, for the proposed model, universal model and bulk stress model, respectively.
- The result shows that the proposed model could potentially be a good alternative for the existing constitutive models.

- To use the proposed model, determination of the four k-coefficients for the subgrade soil under consideration will be required. This could be done by developing correlation equations for k-coefficients with soil properties.

Data Availability Statement

The data that support the findings of this study are available from the LTPP online database at <https://infopave.fhwa.dot.gov/Data/DataSelection> and can be accessed with a reasonable request.

Input data and datasets generated during the current study are also available in the Zenodo repository, <https://doi.org/10.5281/zenodo.7572841>

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