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ABSTRACT

Resilient modulus (M_R) serves as a fundamental material property utilized for characterizing unbound pavement materials in pavement design. While the repeated load triaxial test (RLT) is the standard method to determine M_R, it can be impractical due to the required test facilities and expertise. As a result, various constitutive models have been proposed to predict M_R of soil based on the stress state and soil properties. This study evaluates the performance of ten such constitutive models based on their accuracy in representing the relationship between M_R and stress conditions for RLT test data taken from the long-term pavement performance (LTPP) database. The ten constitutive models consisted of models recommended by various researchers and institutions, including the one proposed by the National Cooperative Highway Research Program (NCHRP) and adopted by the Mechanistic-Empirical Pavement Design Guide (MEPDG). Multi-variable nonlinear curve fitting technique was used to fit the data to the models and obtain model coefficients. The performance of these models was then measured by using the coefficient of determination (R^2), the root mean square error (RMSE) and the mean absolute error (MAE) values as metrics. The results show that the models proposed by NCHRP, Puppala as well as Ni performed best, with Puppala's model having a slight superiority. On the other hand, the models proposed by Rahim and George, Kim, and Moossazadeh and Witczak produced poor results. The results also indicate that models having the general form of NCHRP and expressed in terms of confining pressure and deviator stress perform better. The findings of this study will provide insights into the reliability of existing models and considerations that should be made for developing better alternative models in the future.

Keywords: Constitutive model, resilient modulus, LTPP, nonlinear curve fitting

INTRODUCTION

Resilient modulus (M_R) is a fundamental material property used to characterize unbound pavement materials. Pavement design guides by AASHTO require the use of the resilient modulus for the characterization of unbound pavement material in the design of flexible pavements (AASHTO 1986, 1993, 2002). Resilient modulus is usually determined using a repeated load triaxial test (RLT) on undisturbed or remolded samples (AASHTO 2003). The test is generally conducted for different confining and deviatoric stress values, as M_R is stress dependent. Despite the reliable results, it may not be always feasible to conduct a repeated load triaxial test due to the need for relatively sophisticated test facilities and skilled personnel (Heidaripanah et al. 2017; Khasawneh and Al-jamal 2019; Saha et al. 2018). Different ways for indirect estimation of resilient modulus were proposed by different researchers. The models developed to predict resilient modulus can be generally categorized into two types: constitutive equations and correlation equations. Correlation equations were developed by relating resilient modulus with tests like California bearing ratio (CBR), cone penetration test (CPT) and dynamic cone penetrometer, physical properties like moisture content and dry density, stress states or a combination of them (AASHTO 2003; Ghorbani et al. 2020; Hamed Mousavi et al. 2018; Mohammad et al. 2002, 2007; Sadrossadat et al. 2018, 2020). On the other hand, constitutive models relate resilient modulus with stress invariants. These constitutive models contain unknowns, which can in turn be related to the physical properties of the material (AASHTO 2008; Ghorbani et al. 2020; Liang et al. 2008; Puppala 2008; Saha et al. 2018).

Various constitutive models were proposed by different researchers and institutions which relate M_R with the vertical and/or lateral stress condition of the material. Li et al. (2022) produced a review report which summarizes the commonly used resilient modulus constitutive equations. The most widely used model is the one developed in the National Cooperative Highway Research Program (NCHRP) project 1-28A and adopted by Mechanistic-Empirical Pavement Design Guide (MEPDG) (Andrei et al. 2004; Heidaripanah et al. 2017; Mazari et al. 2016; Saha et al. 2018; Witczak 2003). The list of constitutive models used in this study is provided in Table 1. Models selected were constitutive models that essentially relate stress and strain and were obtained from official design documents, reports and research papers. Correlation equations which have different forms than what is mentioned above were not considered in this analysis.

The reliability of M_R values obtained from these models will depend on how best the models represent the actual relationship between M_R and the stress state of the soil and the accuracy of determining the k coefficients. Regarding determination of k-coefficients, different attempts were made by different researchers to correlate them with soil physical properties, like moisture content, dry density, liquid limit, plasticity index, uniformity coefficient, coefficient of curvature and so on (Elias and Titi 2006; George 2004; S. S. Kim et al. 2019; Malla and Joshi 2008; Nazzal and Mohammad 2010; Sadrossadat et al. 2016; Saha et al. 2018; Yau and Quintus 2002; Zhou et al. 2014).

On the other hand, model performance can be determined by fitting a resilient modulus test data to the model and obtaining k-coefficients (Sadrossadat et al. 2016). Then, performance metrics like coefficient of determination (R^2) and root mean square error (RMSE) can be used to measure the accuracy of the model results. Yau and Quintus (2002) used nonlinear regression analyses on M_R test data from the LTPP database to obtain the performance of NCHRP model. Fedakar (2022) developed neural network models utilizing datasets from LTPP database and other sources, and compared the performance with existing constitutive models. A new data-driven constitutive model was proposed by Demeke et al. (2023) employing data from LTPP database as an input.

Chowdhury and Kassem (2022) developed a regression model for unbound coarse materials which correlates the k-coefficients of the MEPDG constitutive equation with compaction and gradation characteristics of the test aggregates. The performance of the NCHRP model was also evaluated by other researchers (Malla and Joshi 2008; NCHRP 2004).

The objective of this paper is to evaluate the performance of the models listed in Table 1 using M_R test data from the LTPP database and multi-variable nonlinear curve fitting. The result of this analysis will give an insight into the reliability of these models that are currently being recommended for design and will show the comparative accuracy of other developed models. This will be helpful to decide on either sticking to the currently accepted models or other better models should be considered.

MATERIAL AND METHODS

The performances of constitutive models developed for the prediction of M_R were evaluated in this study based on M_R test data obtained from the LTPP database; in this respect, multi-variable nonlinear curve fitting technique was used. The data was first categorized based on the layer type as subgrade and base/subbase. Subgrade data was further divided into two categories based on the soil type as coarse-grained subgrade soil and fine-grained subgrade soil. Model performance analysis was done for five data categories; for all the data, for subgrade soils, for base/subbase materials, for coarse grained subgrade soils and for fine grained subgrade soils, as given in Table 2. Although some models were developed for specific soil types, model evaluation is executed for all cases in order to evaluate their performance out of the condition that they were developed.

The basic steps followed were: data acquisition, data preprocessing, determination of k-coefficients by curve fitting, determination of performance metrics and post processing, including graphing.

Data Acquisition

Data were obtained from the LTPP database in three categories for: course grained materials, fine grained materials and both. A total of 153,702 set of data were collected of which 83,145 refer to coarse grained materials while the rest 70,557 to fine grained materials.

Data Preprocessing

Data preprocessing was carried out in Python using the Pandas library. The first step was removing rows with null values on relevant columns or zero values for M_R, as such value cannot be practical. Duplicate rows were also removed, resulting in a significant reduction of rows (the data were obtained from an expanded table containing duplicate data as required for other purposes). Next, data were filtered based on layer type to get the required categories, and the resulting number of records obtained from the above process is given in Table 2.

Most M_R tests in the LTPP database were executed for 15 combinations of nominal maximum axial stress and confining pressure values (Yau and Quintus 2002). The records were grouped based on key columns as recommended by LTPP using the Pandas library. Grouping was necessary to get the results of tests performed for a unique location at a unique construction stage, as the coefficients of constitutive equations should be determined from the same soil sample tested at different stress states. Groups with less than 15 data points were discarded, as the number of points was not considered to be sufficient to fit the constitutive equations. The total number of groups in each category is given in Table 3 and a typical group is given in Table 4. A graphical representation of a typical group is shown in Fig. 1.

The basic statistical properties of the data in its final form, i.e., after the removal of the records that resulted in groups with data points less than 15, are given in Table 5 while histograms of M_R values are given in Fig. 2.

Curve Fitting

Multi-variable nonlinear curve fitting was done in Python (using the SciPy library) for all groups in each category and for all models given in Table 1, while k-coefficients were determined from these fits. The curve fitting process basically involves fitting the input parameters, i.e., σ_3 and σ_d, and the target value, i.e., M_R, in a specific group into a model and determining the k-coefficients that give the best fit. This process was repeated for each group in all categories. The coefficient of determination (R^2), root mean square error (RMSE) and mean absolute error (MAE) were used as performance indices. These values for each group were calculated using the scikit-learn library in Python. The average of the k-coefficients and metrics obtained for each group in a specific category were computed and were used to compare performances of the models, as given in the next section.

RESULTS AND DISCUSSION

As stated above, curve fitting was applied, and k-coefficients were computed for each group in each category. The average of k-coefficients for each category were calculated and given in Table 6.

Using the computed k-coefficients, M_R values were calculated from each model and performance was measured using these values and actual M_R values. Average R^2, RMSE and MAE values for all categories are presented in Tables 7 to 9, respectively.

All evaluation metrics consistently indicated that the NCHRP method (Witczak 2003), as well as the methods proposed by Puppala et al. (1996), Ni et al. (2002), Witczak and Uzan (1988), and Uzan (1985) performed exceptionally well, with high R^2 values (above 0.9 for most cases) and low RMSE and MAE values. Among them, Puppala's method seems to perform slightly better.

On the other hand, models by Rahim and George (2007), Kim (2004) and Moossazadeh and Witczak (1981) showed poor performance on all metrics. This poor performance was observed even on categories for which these models were developed. It can be observed that the formulation of these models is clearly different from those models that produced relatively good performances. The basic difference is that these models use one coefficient as an exponent for all stresses, while in the other models stresses were treated separately with separate k-coefficients as exponents. Although the model by Seed et al. (1967) was a rather poor method, it gave as good predictions as the NCHRP, Puppala, Ni, Witczak and Uzan for the case of base/subbase. Seed's model is basically similar with NCHRP except the term containing octahedral shear stress (τ_{oct}) in NCHRP. Its good performance on base/subbase materials regardless of not having these term can indicate that these types of materials are not much dependent on octahedral shear stress (τ_{oct}).

It can be seen that models that produced good results were generally those that have similar form with the NCHRP model. These models are expressed as a product of two terms with their respective k-coefficients as exponent, multiplied by a constant term. These two terms are expressed in terms of bulk stress (θ) and octahedral shear stress (τ_{oct}) in NCHRP (Witczak 2003) and Witczak and Uzan (1988), while they are given in terms of confining pressure (σ_3) and deviator stress (σ_d) in Puppala et al. (1996) and Ni et al. (2002). In Uzan (1985), they are given in terms of bulk stress (θ) and deviator stress (σ_d). Of these models, those with σ_3 and σ_d produced slightly better results than others.

It is also worth to note that the performance of models proposed by Uzan (1985) and Witczak and Uzan (1988) produced similar metrics for all cases. This is due to the fact that these two models have the same form and the resulting values of k_2 and k_3 from the curve fitting process was the same. The only difference was the value of k_1, which does not make any difference on the resulting curves.

In addition to the above comparison, the study also evaluated the percentage of data that these models have predicted with an R^2 value above a specific level. Table 10 contains percentage of predictions above R^2 values of 0.95, 0.9, 0.75 and 0.5. For example, Puppala et al. (1996) predicted 85.2 % of all the data with R^2 of more than 0.9 while the prediction by Ni et al. (2002) is 84.8 % for the same case. Generally, the data presented for different categories in Table 10 shows that NCHRP (Witczak 2003), Puppala et al. (1996) and Ni et al. (2002) have the best performances in most of the cases.

CONCLUSIONS

Performance evaluation of constitutive models developed for M_R was done in this study using M_R test data from LTPP database. Multi-variable nonlinear curve fitting was used to fit the data to the models and R^2, RMSE and MAE were used as performance metrics. The major conclusions from the study are:

- Based on R^2, RMSE and MAE values, the models provided by NCHRP (Witczak 2003), Ni et al. (2002), Puppala et al. (1996), Witczak and Uzan (1988), and Uzan (1985) were found to perform better ($R^2 \geq 0.9$ and lowest RMSE and MAE values) for all categories.
- Using the same metrics, the models proposed by Rahim and George (2007), Kim (2004), and Moossazadeh and Witczak (1981) indicated poor performance, including in categories for which these models were specifically developed. The performance of the model by Seed et al. (1967) was also poor, except for the case of base/subbase.
- Analysis of the percentage of M_R test data predicted above a given R^2 value, showed that models provided by NCHRP (Witczak 2003), Puppala et al. (1996) and Ni et al. (2002) are the best among all.
- Models generally resembling the one proposed by NCHRP produced the best results. From these models, those expressed in terms of confining pressure (σ_3) and deviator stress (σ_d) showed slightly better performance than others.
- Finally, the present research work, using big existing data and the multi-variable nonlinear curve fitting technique, further evaluates and elaborates on the performance of various constitutive models developed for M_R, refining the best choice for the optimum constitutive models which should be used and applied for the M_R prediction, and hence provides an additional contribution in correctly determining this fundamental material property used to characterize unbound pavement materials.

DATA AVAILABILITY STATEMENT

All the data used and generated during the study are available online. The data that support the findings of this study are available from the LTPP online database at https://infopave.fhwa.dot.gov/Data/DataSelection and can be accessed with a reasonable request.

The datasets generated during the current study are available in the Zenodo repository with the following citation: https://doi.org/10.5281/zenodo.7157795.

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Table 1: Constitutive models for the prediction of M_R

Researcher	Model	Model applicability
Rahim and George 2007	$M_R = k_1 P_a \left(1 + \frac{\sigma_d}{1 + \sigma_c}\right)^{k_2}$	Fine grained soils
Rahim and George 2007	$M_R = k_1 P_a \left(1 + \frac{\theta}{1 + \sigma_d}\right)^{k_2}$	Coarse grained soils
Kim 2004	$M_R = k_1 \left(\frac{P_a . \sigma_{oct}}{\tau_{oct}{}^2}\right)^{k_2}$	Fine grained soils
NCHRP (Witczak 2003)	$M_R = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{k_3}$	All soil types
Ni et al. 2002	$M_R = k_1 P_a \left(\frac{\sigma_3}{P_a} + 1\right)^{k_2} \left(\frac{\sigma_d}{P_a} + 1\right)^{k_3}$	All soil types
Puppala et al. 1996	$M_R = k_1 P_a \left(\frac{\sigma_3}{P_a}\right)^{k_2} \left(\frac{\sigma_d}{P_a}\right)^{k_3}$	All soil types
Witczak and Uzan 1988	$M_R = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a}\right)^{k_3}$	All soil types

Uzan 1985	$M_R = k_1(\theta)^{k_2}(\sigma_d)^{k_3}$	Granular material
Moossazadeh and Witczak 1981	$M_R = k_1(\sigma_d)^{k_2}$	Fine grained soils
Seed et al. 1967	$M_R = k_1\left(\frac{\theta}{P_a}\right)^{k_2}$	All soil types

Where: M_R = resilient modulus; θ = bulk stress; τ_{oct} = octahedral shear stress; σ_{oct} = octahedral normal stress; P_a = atmospheric pressure (= 101.4 kPa); k_1, k_2 and k_3= coefficients of the tested material; σ_d = deviator stress; σ_3 = confining stress

Table 2: Number of M_R test records by category

Category	Number of Records
All data	42,218
Subgrade data	26,748
Base/subbase data	15,470
Coarse subgrade data	13,722
Fine subgrade data	13,026

Table 3: Number of groups by category

Category		Number of Groups
All data		2,752
Subgrade data		1,741
	Coarse	881
	Fine	860

Base/subbase data	1,011

Table 4: Typical group containing data for a specific sample

Location No	Sample No	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
A10	TS20	41.4	13.7	118
A10	TS20	41.4	27.6	115
A10	TS20	41.4	41.5	107
A10	TS20	41.4	55.1	98
A10	TS20	41.4	68.8	90
A10	TS20	27.6	13.6	113
A10	TS20	27.6	27.1	110
A10	TS20	27.6	40.9	104
A10	TS20	27.6	54.6	97
A10	TS20	27.6	68.4	90
A10	TS20	14	13.6	106
A10	TS20	14	26.6	103
A10	TS20	14	40.4	98
A10	TS20	14	68.7	87
A10	TS20	14	54.6	92

Table 5: Basic statistical properties of the datasets

(a) All data

	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
count	41,280	41,280	41,280
mean	44.211	66.148	106.647
std	35.274	56.026	67.925
min	12.1	11.7	6
25%	20.7	27.6	60
50%	34.4	55.1	85
75%	41.4	68.9	131
max	140.9	287.5	636

(b) Subgrade data

	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
count	26,115	26,115	26,115
mean	27.766	41.352	73.736
std	11.817	20.201	30.789
min	12.1	11.7	6
25%	13.8	27.1	52
50%	27.6	41.3	69
75%	41.3	55.2	90

max	138.1	280.3	296

(c) Base/Subbase data

	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
count	15,165	15,165	15,165
mean	72.530	108.849	163.322
std	43.346	70.423	76.482
min	13.6	13	9
25%	34.5	62	101
50%	68.9	102.8	152
75%	103.5	137.9	214
max	140.9	287.5	636

(d) Coarse subgrade data

	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
count	13,215	13,215	13,215
mean	27.914	41.639	73.588
std	12.347	20.971	29.142
min	12.1	12.1	7
25%	13.8	27.4	54
50%	27.6	41.3	69
75%	41.3	55.2	88

max	138.1	280.3	296

(e) Fine subgrade data

	Confining Pressure (kPa)	Nominal Maximum Axial Stress (kPa)	Average Resilient Modulus (MPa)
count	12,900	12,900	12,900
mean	27.614	41.058	73.888
std	11.246	19.376	32.389
min	13.2	11.7	6
25%	13.8	26.9	51
50%	27.6	41.3	69
75%	41.3	55.2	92
max	42.3	71	257

Table 6: Average values of coefficients of constitutive models obtained from curve fitting

Constitutive Equation	Coefficients	All data	Subgrade	Base/Subbase	Coarse Subgrade	Fine Subgrade
Rahim and George (2007) - Fine Grained	k_1	1.306	0.912	1.984	0.895	0.929
	k_2	-0.251	-0.266	-0.226	-0.237	-0.295
Rahim and George (2007) - Coarse Grained	k_1	0.721	0.541	1.031	0.552	0.529
	k_2	0.277	0.245	0.334	0.219	0.271
Kim (2004)	k_1	128.821	64.567	239.470	67.465	61.597

	k_2	-0.046	0.065	-0.235	0.042	0.087
NCHRP (Witczak 2003)	k_1	0.850	0.848	0.853	0.804	0.893
	k_2	0.486	0.396	0.641	0.480	0.310
	k_3	-0.907	-1.340	-0.162	-1.083	-1.603
Ni et al. (2002)	k_1	64.316	59.955	71.826	52.859	67.223
	k_2	1.271	1.340	1.151	1.601	1.073
	k_3	-0.077	-0.282	0.276	-0.053	-0.516
Puppala et al. (1996)	k_1	1.349	1.003	1.944	1.114	0.889
	k_2	0.333	0.268	0.444	0.322	0.212
	k_3	0.012	-0.064	0.143	-0.011	-0.118
Witczak and Uzan (1988)	k_1	0.615	0.534	0.754	0.544	0.523
	k_2	0.49	0.391	0.660	0.477	0.303
	k_3	-0.133	-0.171	-0.067	-0.141	-0.201
Uzan (1985)	k_1	23.752	32.676	8.383	21.324	44.306
	k_2	0.49	0.391	0.660	0.477	0.303
	k_3	-0.133	-0.171	-0.067	-0.141	-0.201
Moossazadeh and Witczak (1981)	k_1	72.452	98.054	28.365	82.263	114.230
	k_2	0.118	-0.060	0.425	-0.005	-0.116
Seed et al. (1967)	k_1	75.895	70.537	85.121	68.997	72.115
	k_2	0.378	0.251	0.597	0.361	0.138

Table 7: Average R^2 values of the constitutive models

Constitutive Equation	All data	Subgrade	Base/Subbase	Coarse Subgrade	Fine Subgrade
Rahim and George (2007)					
- Fine Grained	0.36	0.54	0.06	0.42	0.66
- Coarse Grained	0.36	0.52	0.08	0.40	0.64
Kim (2004)	0.37	0.39	0.33	0.25	0.53
NCHRP (Witczak 2003)	0.93	0.90	0.98	0.90	0.91
Ni et al. (2002)	0.93	0.91	0.97	0.91	0.91
Puppala et al. (1996)	0.94	0.91	0.99	0.92	0.91
Witczak and Uzan (1988)	0.93	0.89	0.98	0.90	0.89
Uzan (1985)	0.93	0.89	0.98	0.90	0.89
Moossazadeh and Witczak (1981)	0.42	0.29	0.65	0.19	0.39
Seed et al. (1967)	0.62	0.42	0.96	0.57	0.27

Table 8: Average RMSE values of the constitutive models

Constitutive Equation	All data	Subgrade	Base/Subbase	Coarse Subgrade	Fine Subgrade
Rahim and George (2007)					
- Fine Grained	25.7	7.1	57.8	8.8	5.3
- Coarse Grained	25.6	7.2	57.2	9.0	5.5

Kim (2004)	22.8	8.2	47.9	10.1	6.3
NCHRP (Witczak 2003)	4.1	2.7	6.5	3.0	2.3
Ni et al. (2002)	4.8	2.6	8.6	2.9	2.3
Puppala et al. (1996)	3.6	2.6	5.4	2.8	2.5
Witczak and Uzan (1988)	4.0	2.9	6.1	3.0	2.8
Uzan (1985)	4.0	2.9	6.1	3.0	2.8
Moossazadeh and Witczak (1981)	18.0	9.0	33.6	10.5	7.5
Seed et al. (1967)	7.9	7.5	8.5	6.9	8.1

Table 9: Average MAE values of the constitutive models

Constitutive Equation	All data	Subgrade	Base/Subbase	Coarse Subgrade	Fine Subgrade
Rahim and George (2007)					
- Fine Grained	22.0	5.9	49.7	7.4	4.3
- Coarse Grained	21.8	6.2	48.7	7.7	4.7
Kim (2004)	20.0	7.0	42.4	8.7	5.3
NCHRP (Witczak 2003)	3.3	2.2	5.2	2.4	1.9
Ni et al. (2002)	3.9	2.1	7.0	2.4	1.9
Puppala et al. (1996)	2.9	2.1	4.3	2.2	2.0
Witczak and Uzan (1988)	3.2	2.3	4.8	2.4	2.3
Uzan (1985)	3.2	2.3	4.8	2.4	2.3

Moossazadeh and Witczak (1981)	15.2	7.6	28.2	8.9	6.3
Seed et al. (1967)	6.4	6.2	6.9	5.6	6.8

Table 10: Percentage of R^2 values by range

(a) All data

Constitutive Equation	R^2 greater than 0.95 (%)	R^2 greater than 0.9 (%)	R^2 greater than 0.75 (%)	R^2 greater than 0.5 (%)
Rahim and George (2007)				
- Fine Grained	2.1	9.2	24.3	35.6
- Coarse Grained	0.7	6.2	21.5	34.5
Kim (2004)	1.3	5.1	13.7	25.8
NCHRP (Witczak 2003)	66.5	81.5	94.7	98.0
Ni et al. (2002)	69.8	84.8	95.4	98.2
Puppala et al. (1996)	68.6	85.2	95.4	98.3
Witczak and Uzan (1988)	60.0	80.7	94.0	98.1
Uzan (1985)	60.0	80.7	94.0	98.1
Moossazadeh and Witczak (1981)	0.3	1.2	10.8	49.3
Seed et al. (1967)	31.9	40.2	51.8	62.9

(b) Subgrade

Constitutive Equation	R^2 greater than 0.95	R^2 greater than 0.9	R^2 greater than 0.75	R^2 greater than 0.5
Rahim and George (2007)				
- Fine Grained	3.4	14.6	38.0	55.7
- Coarse Grained	1.0	9.8	33.7	53.8
Kim (2004)	2.0	8.0	21.4	37.9
NCHRP (Witczak 2003)	51.6	72.6	91.9	97.0
Ni et al. (2002)	56.6	77.1	93.1	97.3
Puppala et al. (1996)	53.2	77.6	93.1	97.4
Witczak and Uzan (1988)	40.2	70.6	90.8	97.1
Uzan (1985)	40.2	70.6	90.8	97.1
Moossazadeh and Witczak (1981)	0.2	1.2	9.2	25.6
Seed et al. (1967)	3.1	10.4	25.5	42.1

(c) Base/Subbase

Constitutive Equation	R^2 greater than 0.95	R^2 greater than 0.9	R^2 greater than 0.75	R^2 greater than 0.5
Rahim and George (2007)				
- Fine Grained	0.0	0.0	0.7	1.0
- Coarse Grained	0.0	0.0	0.7	1.3
Kim (2004)	0.0	0.1	0.5	5.0

NCHRP (Witczak 2003)	92.2	96.9	99.6	99.8
Ni et al. (2002)	92.7	98.0	99.3	99.7
Puppala et al. (1996)	95.2	98.4	99.5	99.8
Witczak and Uzan (1988)	94.1	98.1	99.5	99.8
Uzan (1985)	94.1	98.1	99.5	99.8
Moossazadeh and Witczak (1981)	0.4	1.1	13.5	90.2
Seed et al. (1967)	81.5	91.5	97.1	98.8

(d) Coarse subgrade

Constitutive Equation	R^2 greater than 0.95	R^2 greater than 0.9	R^2 greater than 0.75	R^2 greater than 0.5
Rahim and George (2007)				
- Fine Grained	1.9	9.2	21.8	37.6
- Coarse Grained	0.7	5.7	18.2	35.4
Kim (2004)	0.3	2.0	9.0	20.8
NCHRP (Witczak 2003)	48.1	69.7	92.2	97.2
Ni et al. (2002)	56.9	76.1	93.3	97.5
Puppala et al. (1996)	58.9	78.7	93.6	97.5
Witczak and Uzan (1988)	48.9	72.3	91.6	97.3
Uzan (1985)	48.9	72.3	91.6	97.3
Moossazadeh and Witczak (1981)	0.0	0.3	3.4	12.6

Seed et al. (1967)	5.0	17.5	40.4	60.5

(e) Fine subgrade

Constitutive Equation	R^2 greater than 0.95	R^2 greater than 0.9	R^2 greater than 0.75	R^2 greater than 0.5
Rahim and George (2007)				
- Fine Grained	4.9	20.1	54.7	74.2
- Coarse Grained	1.4	14.0	49.5	72.7
Kim (2004)	3.7	14.1	34.1	55.3
NCHRP (Witczak 2003)	55.2	75.6	91.6	96.9
Ni et al. (2002)	56.3	78.3	92.9	97.1
Puppala et al. (1996)	47.3	76.5	92.4	97.2
Witczak and Uzan (1988)	31.3	69.0	89.9	97.0
Uzan (1985)	31.3	69.0	89.9	97.0
Moossazadeh and Witczak (1981)	0.5	2.1	15.1	38.8
Seed et al. (1967)	1.2	3.1	10.2	23.3

Figure 1

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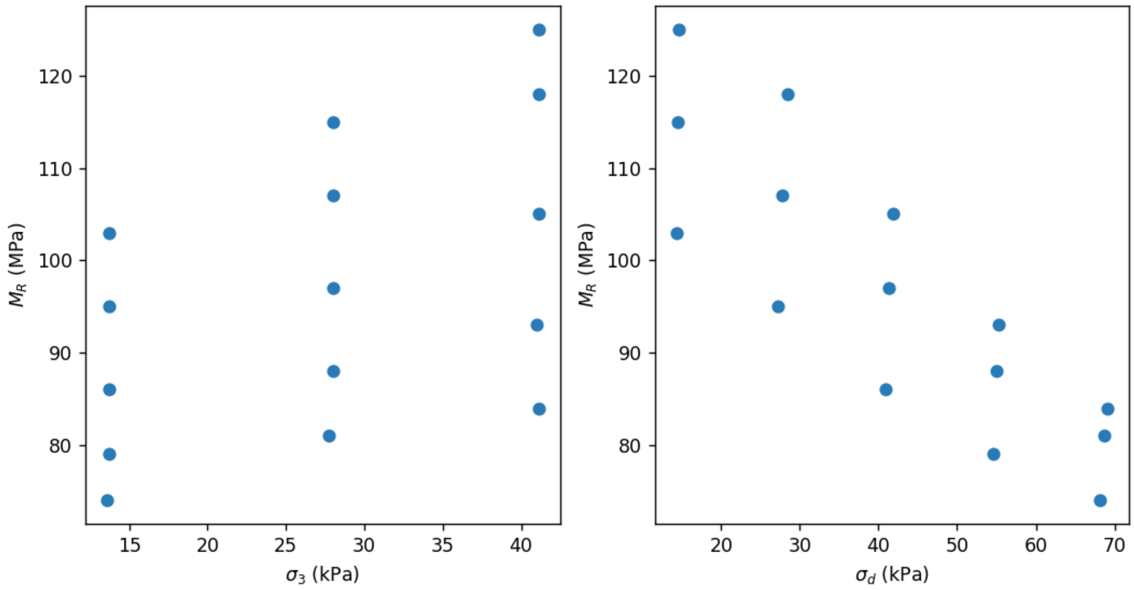

Figure 2 (a)

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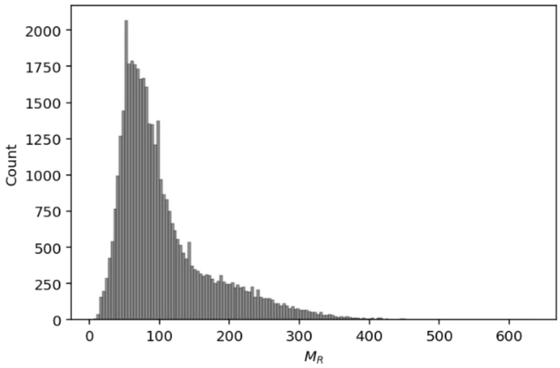

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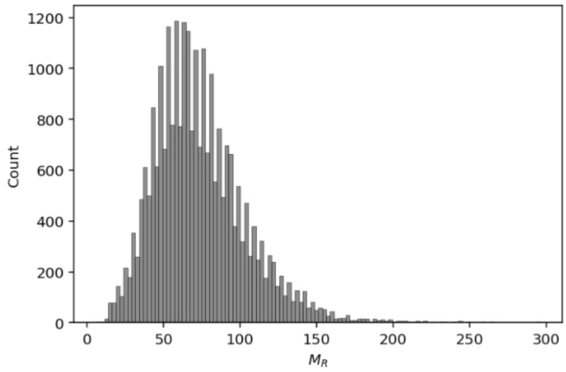

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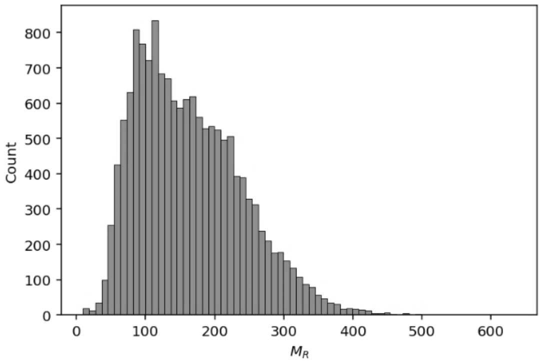
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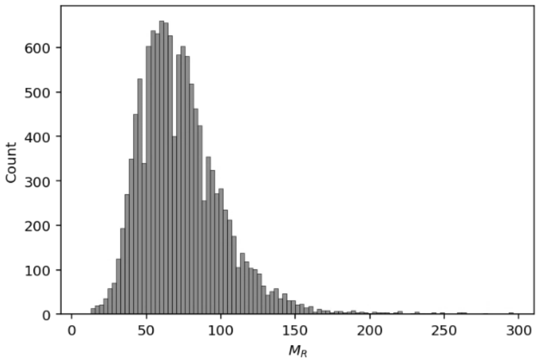

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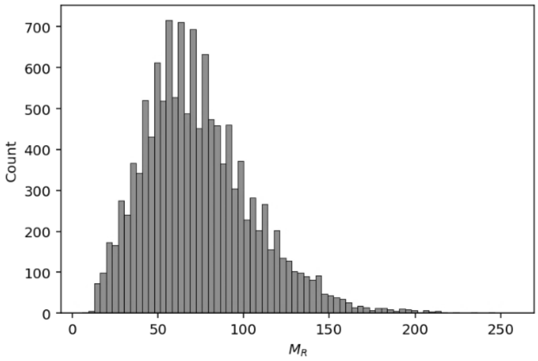

Figure 1: Plot of a typical group

Figure 2: Histogram of M_R values in MPa. (a) all materials, (b) subgrade materials, (c) base and subbase materials, (d) coarse subgrade materials, (e) fine subgrade materials